Problem

Federico Castillo

May 20, 2019

1 Background

We will deal mostly with 2-dimensional polygons.

Definition 1. We say a polygon Q is a *deformation* of polygon P (denoted $Q \prec P$) , if every *edge* of Q is parallel to an edge of P.
We say P and Q are normally equivalent if $Q \prec P$ and the set of edge directions is the same.

Informally, we can get Q from P by moving the vertices while keeping the edge directions and possibly collapsing some of them. In fact, Q is normally equivalent to P if no edge collapsed.

Definition 2. Fixing a polygon P there is a finite number of deformations of P (up to norm. equivalence). This equivalence classes form a poset $\mathcal{L}_P$ under the $\prec$ relation.

Example 1. *For instance, if P is the pentagon below the complete poset looks as follows.*

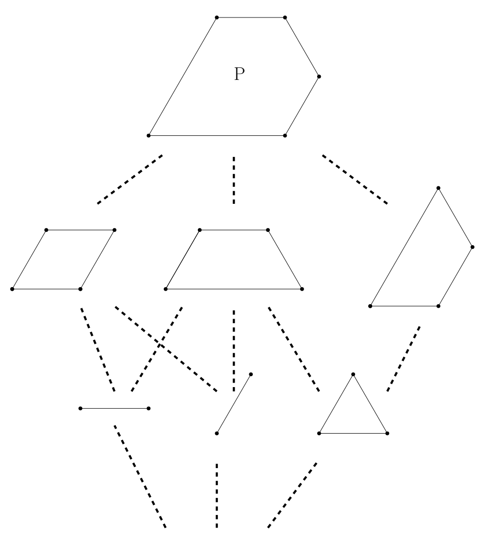

Figure 1: Poset for a pentagon.

The main goal is to understand this poset when the initial polygon is a *regular* n-agon. Being regular is of consequence, the poset for a regular pentagon has more elements than the previous example.

2 Questions

We focus on P_n regular n-agon.

Poset It is a fact not too hard to show in this particular example $\mathcal{L}_{P_n}$ is the face lattice of a $(n-3)$ dimensional polytope. The main motivation is to understand the combinatorics of the polytope. For this it is not necessary to understand anything about polytopes, the combinatorial information is in the poset.

Polytope It is not hard to see that the polytope has n facets, so it may be easier to study the dual, where now we have n vertices in $n-3$ dimension. This situation is suitable for gale diagrams and oriented matroid duality.

f-vector Compute the rank function of the poset. Here is where being a polytope helps. The dual of this polytope has n vertices in dimension $n-3$ and it is almost neighborly, this gives for free most of the $f-$vector. One only needs to compute extra few middle entries and then through h-vectors and Dehn-Sommerville one can potentially compute the whole thing.

General case How different is for general polygons?

Group action Notice how the symmetries of P, in this case the dihedral groups D_n act on each level of the poset. In this direction one can consider some directions: 1. Study the *quotient*. 2. Study the action of D_n on the homology groups of the order complex.